

## Three-Tier Mathematics Intervention Model

**Special Education Research Project (SERP) Mathematics:**  
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**This model is based on the 3-Tier Reading Model**  
**Vaughn Gross Center for Reading & Language Arts**  
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### Advance Organizer

- Goal: To share lessons learned about 3-Tier Math Model development
- Focus on assessment
- Focus on intervention
- Background for new participants
- Questions midway and at the end

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### Core Features of RTI

1. High quality classroom instruction that is research-based -
  - \* Can be determined by examining the research base of the programs being used
  - \* Can be assessed by comparing students' learning rates and achievement across same grade level classrooms
2. Universal screening on academics and behavior -
  - \* Criteria are used to judge the learning and achievement of all students
3. Continuous progress monitoring -
  - \* Data can be used to determine students who are not reaching benchmark



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## Core Features of RTI

4. Research-based interventions -

- \* Possibilities include standard protocol procedures that have been validated
- \* May occur 8 - 12 weeks in length
- \* Designed to be more intensive

5. Fidelity measures -

- \* Documentation that procedures are being implemented accurately as described and validated through research (observational checklist of critical teaching behaviors)



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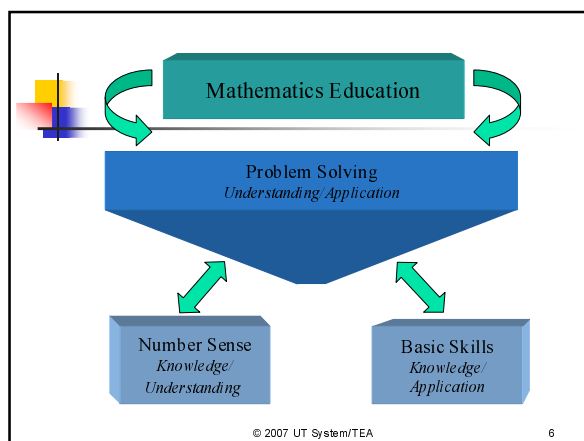
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## NCTM Curriculum Focal Points & Connections, Sept. 2006

<http://www.nctm.org/focalpoints/downloads.asp>

**Kindergarten:**  
 Number & Operations: Representing, comparing, and ordering whole numbers and joining and separating sets (Geometry, Measurement)

**First Grade:**  
 Number & Operations & Algebra: Developing understandings of addition & subtraction and strategies for basic addition facts and related subtraction facts

Number & Operations: Developing an understanding of whole number relationships including grouping in tens & ones (Geometry)

**Second Grade:**  
 Number & Operations: Developing an understanding of the base-ten numeration system and place-value concepts

Number & Operations & Algebra: Developing quick recall of addition facts and related subtraction facts & fluency with multidigit addition and subtraction (Measurement)

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### What is the 3-Tier Mathematics Intervention Model?

- Is an assessment & intervention model designed to meet the instructional needs of students in grades K - 2 who are identified as struggling with mathematics
- Provides a framework for providing instruction and using assessment data to inform decision-making
- Is a response-to-intervention model (developing)
- Focuses on standards-based intervention (number & operation, algebra, problem solving [computation, time, money])

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### What are the Components of the 3-Tier Mathematics Intervention Model?

- Tier 1: Core classroom instruction for all students (45-90 minutes-observed in K-4)
- Tier 2: Intervention for approximately 10 - 30% of identified students 20 minutes-10 - 12 weeks - 3 to 4 days a week); Includes differentiated instruction in number and operation; Includes explicit instruction in small, homogeneous groupings
- Tier 3: Intensive intervention for approximately 5-8% of identified students (may include special education students; probably another 20 minutes?)

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### Core Educational Problem: Assessment

- Limited availability of technically adequate measures for identification and to monitor response to intervention of Tier 2 students in the primary grades
- Need to develop technically adequate measures for early mathematics number, operation, and quantitative reasoning skills and concepts
- Measures can contribute to an understanding of predictors of early mathematics performance, inform mathematics instructional decisions, and change mathematics outcomes for students who are at risk for mathematics difficulties
- Need to establish benchmarks

Chen, Clarke, Baker, O'Brien and Brown & Kato 2007 UT System/TEA 10

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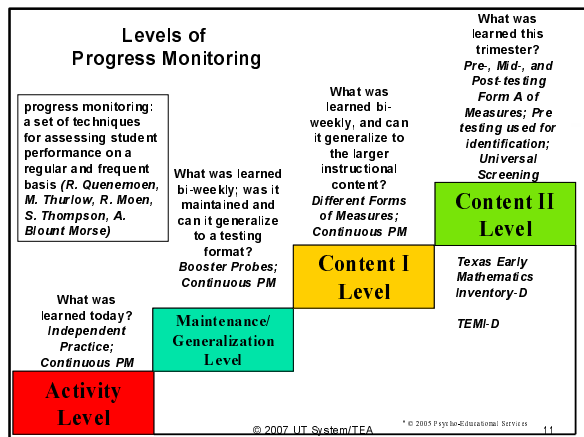
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Missing Numbers: Independent Practice 100-420

Scoring: Count each box as correct or error even if there is more than one response in the box.

Do Column A first				Do Column B second			
210	212			307			310
412	414			201			204
101	102			413			416
211	213			308	309		
309	311			117	119		
113	115			404	406		
301	303			217	219		
218	219			310	311		
109	111			401	403		
406	407			210	212		

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Activity Level Weekly Progress Monitoring

Students	Day 1	Day 2	Day 3	Day 4
Amanda	WYN: 6/6 MC: 6/6 PV: 3/4	Date: 10/10 WPS: 7/8 NS: 8/8		
Cathy	Absent			
Barbara	WYN: 6/6 MC: 6/6 PV: 3/4			
Deanna	WYN: 6/6 MC: 5/6 PV: 4/4			
Rockelle				

Behavior: \_\_\_\_\_

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### Probes and Clinical Interviews

9 <u>+9</u>	6 <u>+6</u>	4 <u>+4</u>	7 <u>+7</u>	5 <u>+5</u>
1 <u>+1</u>	8 <u>+8</u>	3 <u>+3</u>	2 <u>+2</u>	1 <u>+1</u>
8 <u>+8</u>	7 <u>+7</u>	2 <u>+2</u>	4 <u>+4</u>	1 <u>+1</u>
3 <u>+3</u>	5 <u>+5</u>	6 <u>+6</u>	9 <u>+9</u>	0 <u>+0</u>

Maintenance/  
Generalization  
Level Progress Monitoring

Specific Instructional Content

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### PM Measures



**Magnitude Comparisons**  
K: 0-20, bigger #, same  
1: 0-99, smaller #, same  
2: 0-999, less, equal

1	0	9	4	1	1	14	10
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### PM Measures

**Numerical Sequences**  
K: 0-20  
1: 0-99  
2: 0-999

1	2	___	16	___	18	___	81	82
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Place Value

PM Measures

1: 1-99

2: 1-999

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PM Measures

Addition/Subtraction  
Combinations, to/from 18  
(1 & 2 only)

6

7

2

11

5

0

5

- 2

+ 1

- 1

- 8

+ 0

+ 3

- 2

4

13

1

12

13

6

0

+ 3

- 6

+ 0

- 7

- 4

- 0

+ 2

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PM Measures

18

8

15

11

16

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Numeral Naming (K Only)

0-20

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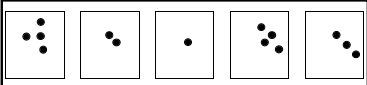

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## PM Measures

**Quantity Recognition (K Only)**  
1-6

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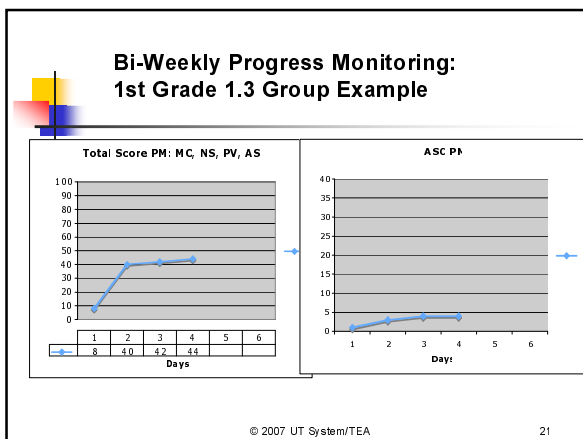
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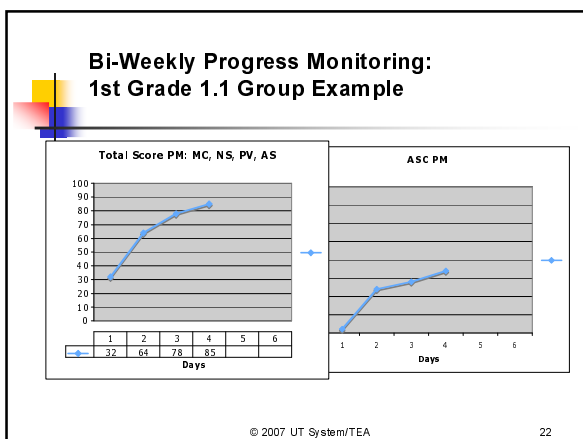
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## ???????Question Time???????



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## Core Educational Problem: Intervention

- Limited evidenced-based interventions demonstrating efficacy for improving mathematics performance in early mathematics skills and concepts
- Need to develop, refine, and evaluate interventions to teach students in kindergarten, first, and second grades who have been identified as Tier 2 for mathematics difficulties
- Number and operations is cited as the most important area of NCTM's (2000) *Principles and Standards for School Mathematics* (Clements & Sarama, 2004)
- Automaticity is identified as "desirable" at an early stage of formal mathematics education (Cumming & Blöme, 1998)

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## Tier 1

- Balanced approach to mathematics instruction
- Opportunities for students to engage in meaningful practice
  - Partner Math
  - CAI
- Mathematically enriched environment that makes math visible
  - Vocabulary Words
  - Abstract Symbols
  - Manipulatives and Tools
  - Calculators
- Explicit instruction to teach procedural knowledge (basals)
- Questioning strategies that require explanations and descriptions
- Progress monitoring
- Problem solving
- Instructional adaptations

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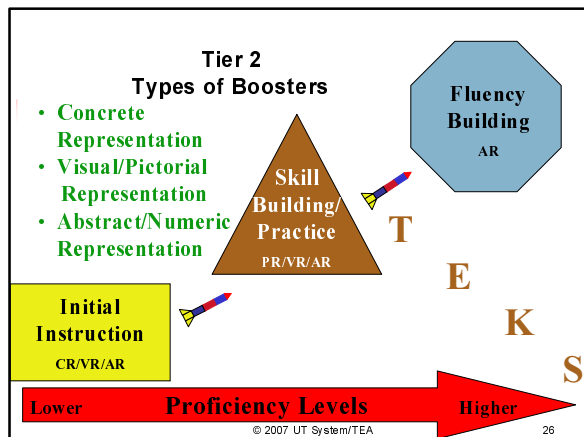
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**Effective Instruction  
for Booster Sessions**

- Framing the lesson\*
- Previewing
- Modeling w/think alouds
- Guided practice
- Independent practice
- Checking for understanding
- Error correction and feedback
- Progress monitoring

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**Quality Implementation Indicators: Fidelity**

Teacher Behavior	Most of the Time	Some of the Time	Rarely	Not at all
<b>Intervention</b>	3	2	1	0
Teacher follows script/protocol to ensure fidelity of implementation.				
Teacher implements each step/modeling CP, P, or I sufficiently to ensure fidelity of implementation.				
Teacher implements self-correction following P to ensure students learn IC.				
<b>Instruction</b>				
Teacher monitors and provides.				
Teacher provides corrective feedback immediately as needed.				
Teacher talk is kept to a minimum and is characterized with short requests "What are we?" "Are you?"				
Teacher engages students throughout lesson with response that is verbal, written or hand-drawn.				
Teacher models using "think aloud."				

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### Quality Implementation Indicators: Fidelity

Student Behavior Management				
Teacher focuses attention on appropriate behavior (e.g., reinforcement).				
Teacher provides immediate and appropriate feedback on math readiness.				
Teacher intervenes quickly to stop inappropriate behavior.				
Teacher reduces behavior.				
Lesson Management				
Teacher adheres to time & use of time to manage time.				
Teacher has materials management system that minimizes time to get materials.				

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### The Characteristics of Students with Mathematics Disabilities (MD)

- Difficulties with base ten concepts
  - Place value difficulties (concept of 0, number of digits in a numeral to show place value)
- Procedural difficulties
  - Immature strategies use (count all)
  - Errors in math problem execution
- Memory problems
  - Poor long-term memory retrieval skills (recalling procedural knowledge and factual knowledge)
  - Working memory deficits: facts and number recognition (teens, reversals, confusing similar looking numerals)
- Visual/spatial deficits
  - Weak visual/spatial representations (numbers are immaturely made, pencil grip, difficulty writing in spaces/boxes, numbers are shaky)
- Low number sense
  - Number magnitude comparison confusion
  - Poor number naming, writing, sequencing

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### Procedures & Features of Tier 2 Intervention: What We've Learned

(1) Groupings: homogeneous grouping with 2 - 5 students per group; 3 levels within in grade level .1, .2, .3  
Issue: group size should not exceed 5

(2) Duration: 4 times per week for 20 minutes  
Issue: 20 minutes (competing for instructional time)

(3) Lesson Design: sequential & mixed (instructional content), scaffolded (adaptations), scripted interventions; explicit, strategic, "think aloud;" error correction; factual and procedural learning  
Issue: making sure include mixed and review

(4) Instructional Content: IC ranges focusing on difficult numbers (teens, 3-digit #'s with 0 place holder or teens); mathematics vocabulary; (e.g., greater than/less than); number/operation, algebra, problem solving  
Issue: teens are difficult (language); visual representations/keyword & pegword

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## Procedures & Features of Tier 2 Intervention: What We've Learned

(5) Representations: physical (concrete), visual (pictorial), abstract (numbers)  
 Issue: ensuring enough of the 3 levels: 2 weeks of instruction on same content across representations

(6) Materials: number charts (100s), 5- and 10-frames, counters, cubes, number lines (horizontal/vertical), base-ten materials, dot cards  
 Issue: controlling for the number of materials within the 20 minute lesson and keeping student engaged

(7) Stages of Learning-acquisition (control materials); generalization: IP to probes to TEMI-D  
 Issue: engaging students in a similar response to monitor progress

(8) Progress monitoring: activity level-daily (independent practice: 1-2 minutes depending on grade level and difficulty of content); bi-weekly (probes or TEMI-D) Issue: for RTI moving students out, and establishing benchmarks

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Preview  
*SWs are going to show and write numbers in different ways.*

### Modeling (My Turn)

1. Make a number using the flats, rods, and units from the Instructional Content on PV chart.
2. Explain that you will put the number in each place on the place value chart to represent the quantity in each place. Example: I have 1 hundred so I put a 1 in the hundreds place.
3. Have students count each place value and say the number.
4. Write the number on the wipe board.

### Guided Practice (Our Turn)

WEEK 6A: DAY 1 IC MIXED

1. Make a number using the flats, rods, and units from the Instructional Content on PV chart.
2. Have students count each place value and say the number.
3. Write the number on the wipe board. Have students write the number on wipe board.
3. Put PV small flashcards for instructional content (just pick 4 cards one of which has the answer) on table. Have students point to card that depicts the number made.
4. Repeat steps 1-3 for different IC content.

WEEK 6A: DAY 3 IC MIXED

1. Make a number using the flats, rods, and units from the Instructional Content on PV chart.
2. Have students count each place value and say the number.
3. Show the PV large flashcard for the model. Have students write the number on their wipe boards.
4. Repeat steps 1-3 for different IC content.

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## Tier 3

- More intensive in terms of time and grouping
- Explicit and systematic
- Representations
- Program
- Individualized? Standard protocol?



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## Implementation Questions

- **How do you know if Tier 1 (core) instruction is not working?** Assess all students 3 times a year; students not showing appropriate progress may qualify for Tier 2 instruction (cut score < 16 - 25th percentile)
- **What should Tier 2 instruction look like?** See previous Tier 2/3 instruction slide for example
- **How do we know if Tier 2/3 instruction is working (are students responding to instruction)?**  
Progress monitor students regularly

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## Implementation Questions

- **How long is Tier 2 instruction implemented?** 10-12 weeks; reassess if progress move to Tier 1; if limited progress conduct another 10-12 weeks of Tier 2; if no progress consider Tier 3
- **How often should we progress monitor students?** Tier 2, bi-weekly; Tier 3 - weekly - recommendation
- **How do I assess fidelity?** Use a checklist containing expectations for Tier 2 to decide if intervention practices are being used with fidelity; see Instructional Decision Making booklet - administrator's pages

[http://www.texasreading.org/utoria/materials/serp\\_prereferal\\_booklet.asp](http://www.texasreading.org/utoria/materials/serp_prereferal_booklet.asp)

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## What Are Helpful Resources?

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- Special issue of JLD-July issue-Gersten & Jordan (Bryant)
- Special issue: RASE
- Special issue: LDQ

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